



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 26

The Barbell Strategy - 1

J P Singh

Department of Management Studies



DURATION MATCHING THROUGH CASH FLOW MATCHING

- Cash flow matching is the simplest but least flexible approach to immunizing.
- For a single liability, buy a zero-coupon bond with par and maturity matching the liability.
- With no cash flows to reinvest or bonds to sell, there is no cash flow or price risk.
- The bonds must also be default free.



- **Because a single liability is effectively a zero-coupon liability, its time to payment is also its Macaulay duration.**
- **Thus, a cash flow matched immunized portfolio will match the Macaulay durations of the assets and liabilities.**

DURATION MATCHING (GENERAL)

- Match the present values of assets & liability.
- Match the duration of the assets & liability.
- Match the convexity of the assets & liability to minimize dispersion.



BARBELL STRATEGY



BARBELL STRATEGY: EXAMPLE

- Compare two portfolios I & II in respect of their duration & convexities:
- Portfolio I: Equal money weights (INR 1,000) of 1 year and 9 year ZCBs.
- Portfolio II: An investment of INR 2,000 in 5-year ZCB.
- All the bonds are trading at a YTM of 15% p.a.



DURATION PORTFOLIO I

TIME	CASH FLOW C(T)	TC(T)	DISC TC(T)
0	-2000	0	0
1	1150	1150	1000
9	3517.88	31660.92	9000
IRR	0.150000121		10000
DURATION			5

DURATION OF PORTFOLIO II IS ALSO 5-YEARS SINCE IT CONSISTS OF ONLY 5 YEAR ZCBS.

CONVEXITY

TIME	CASH FLOW C(T)	T(T+1)C(T)	DISC T(T+1)C(T)
0	-2000	0	0
1	1150	2300	1999.999789
9	3517.88	316609.2	90000.00949
IRR	0.150000121		92000.00928
CONVEXITY			23.000000232

TIME	CASH FLOW C(T)	T(T+1)C(T)	DISC T(T+1)C(T)
0	-2000	0	0
5	4022.71	120681.3	60000
IRR	0.14999975		60000
CONVEXITY			15

CONCLUSION

- The example shows that Portfolio I consisting of equal money weights (INR 1,000) of 1 year and 9 year ZCBs has:
- (i) same duration but
- (ii) higher convexity
- as compared to portfolio II that has an investment of INR 2,000 in 5-year ZCB.
- All the bonds are trading at a YTM of 15% p.a.
- The portfolio I strategy is called the barbell strategy and may be employed to enhance convexity of a bond investment.

PORTFOLIO DISPERSION VS CONVEXITY

- Portfolio dispersion is computed as the weighted average variance of when each cash flow is received around portfolio duration. Duration is just the weighted average of when all the cash flows are received.

$$\text{Convexity}_{\text{mod}} = \frac{D_{\text{Mac}}^2 + D_{\text{Mac}} + \text{Dispersion}}{(1 + \text{Periodic Portfolio IRR})^2}$$

EFFECT OF YIELD CURVE CHANGES ON IMMUNIZED LIABILITY



IMMUNIZATION OF A SINGLE LIABILITY BY BARBELL PORTFOLIO

- Assume that we are using a barbell (B) to immunize a single liability (sometimes called a bullet) due in five years.
- The durations of the barbell (B) and liability D are initially matched at the 5.0 medium (M) duration of the liability,.
- Barbell portfolio convexity exceeds convexity of the single cash flow liability.

- We can create an extreme situation to demonstrate the issues.
- Assume that the bullet liability (D) is a ZCB of maturity $M=5$ years.
- Assume that the barbell portfolio is made up of two bonds with a shorter ($S=1$ year) and longer ($L=9$ years) duration than the liability duration (M).
- The initial value of the barbell equals the discounted (at portfolio IRR) PV of the liability.
- This describes a barbell portfolio strategy, concentrating the assets in longer and shorter duration around the liability's single (bullet) duration.

PARALLEL SHIFTS IN YIELD CURVE

- If the yield curve shifts up or down in parallel fashion, the barbell portfolio results will slightly exceed the amount required to pay the future liability.
- If the yield price curve was linear, duration matching would have led to exact meeting the future liability need.

- However, the yield price curve is not linear. The positive convexity of the assets will lead them to outperform duration results alone for large parallel shifts in the curve.
- While higher the convexity of the barbell, greater is the outperformance of the barbell over the bullet, greater will also be the variability or dispersion of the barbell side.
- Recall the dispersion of cash flows and thus the convexity of the two-bond portfolio must exceed that of the single-point liability.

PARALLEL INCREASE IN YIELDS

- We have: $\frac{dP}{P} = -D \frac{dy}{1+y} + C \left(\frac{dy}{1+y} \right)^2$
- If we ignore convexity, for a large parallel increase in the curve ($dy > 0$), the percentage change in price captured by the duration term cancels out between the barbell & bullet as the immunizing barbell duration matches the liability duration.

- However, the convexity correction $C \left(\frac{dy}{1+y} \right)^2$ (which is always positive for positive convexity irrespective of the sign of dy) will be greater for the immunizing portfolio since the (barbell) immunizing portfolio has larger convexity than the bullet liability being immunized.
- Thus, the immediate decrease in portfolio value will be less than the decrease in the PVL due to the positive convexity effect.

- With the parallel increase, the new portfolio IRR will increase by basically the same amount as the increase in discount rate for the PVL.
- In other words, the future rate of increase in assets and liability are still the same, but starting from a new PVA that is relatively higher than the PVL, the FVA will exceed the FVL.

EXAMPLE

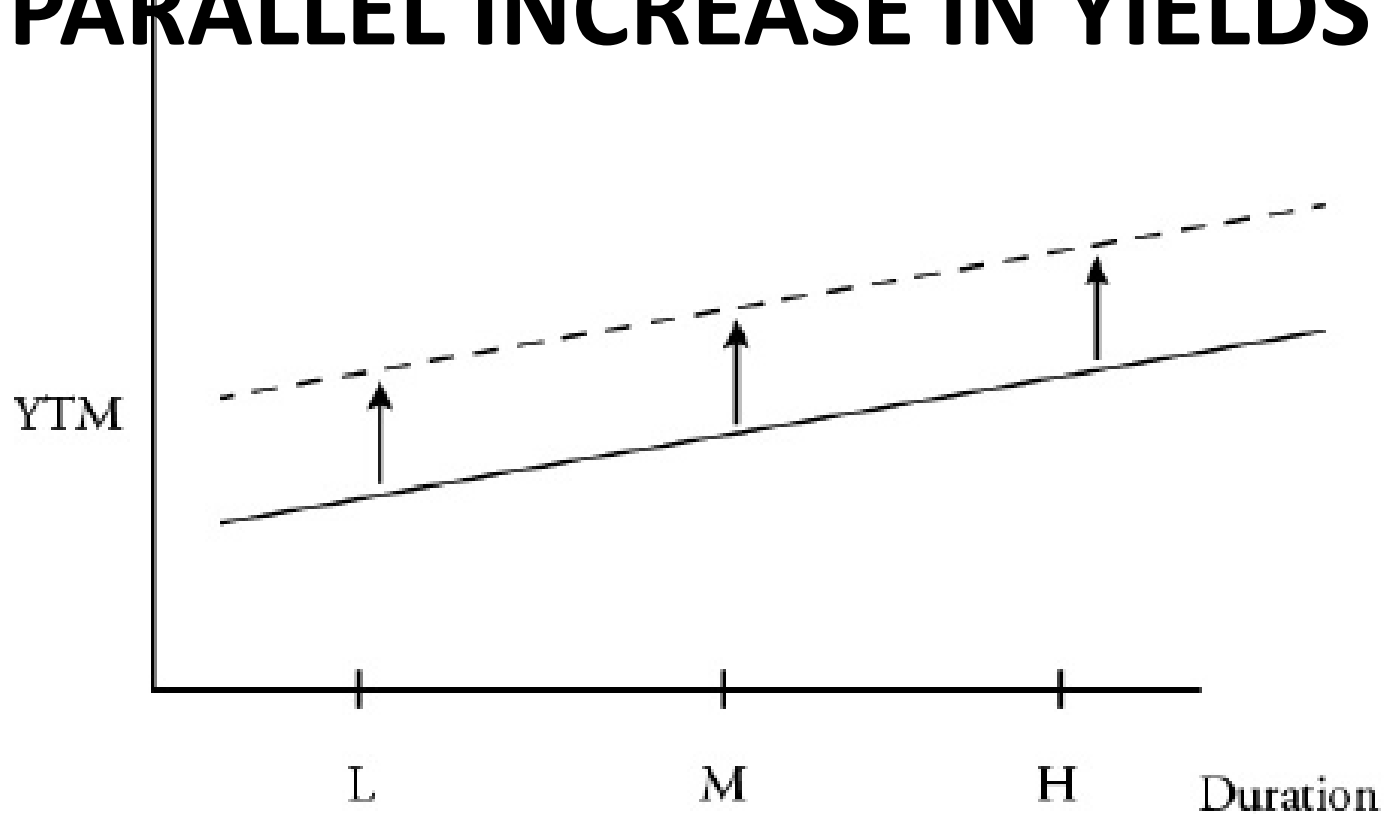
- Consider the barbell strategy consisting of two bonds S & L with equal money weights (INR 1,000) of 1 year and 9 year ZCBs for immunizing a ZCB liability M of INR 4,022.71 due at $t=5$ years. All the bonds are trading at a YTM of 15% p.a. Assume that there is a instantaneous parallel upward shift of the yield curve by 2%. Evaluate the performance of the strategy.

SOLUTION

- $PVL(P_M^0) = \frac{4022.71}{1.15^5} = 2000$
- $PVA(P_{BARBELL}^0) = 1000 + 1000 = 2000$
- After the interest rate shift by 2% from 15% to 17%, we have:
- $PVL(P_M^1) = \frac{4022.71}{1.17^5} = 1834.80$
- $P_S^1 = \frac{1,150}{1.17} = 982.91$; $P_L^1 = \frac{3517.88}{1.17^9} = 856.27$
- $P_{BARBELL}^1 = P_S^1 + P_L^1 = 1839.18$
- Clearly $P_{BARBELL} > P_M$

- In this problem, the initial IRR of both the barbell & the liability is 15% and the shift is 2% upwards uniformly. Hence, value of barbell & liability will both grow at 17%.
- Thus, at maturity of liability (t=5 years)
- Value of liability = $1834.89 \times 1.17^5 = 4022.90$
- Value of barbell = $1839.18 \times 1.17^5 = 4032.30$

PARALLEL INCREASE IN YIELDS

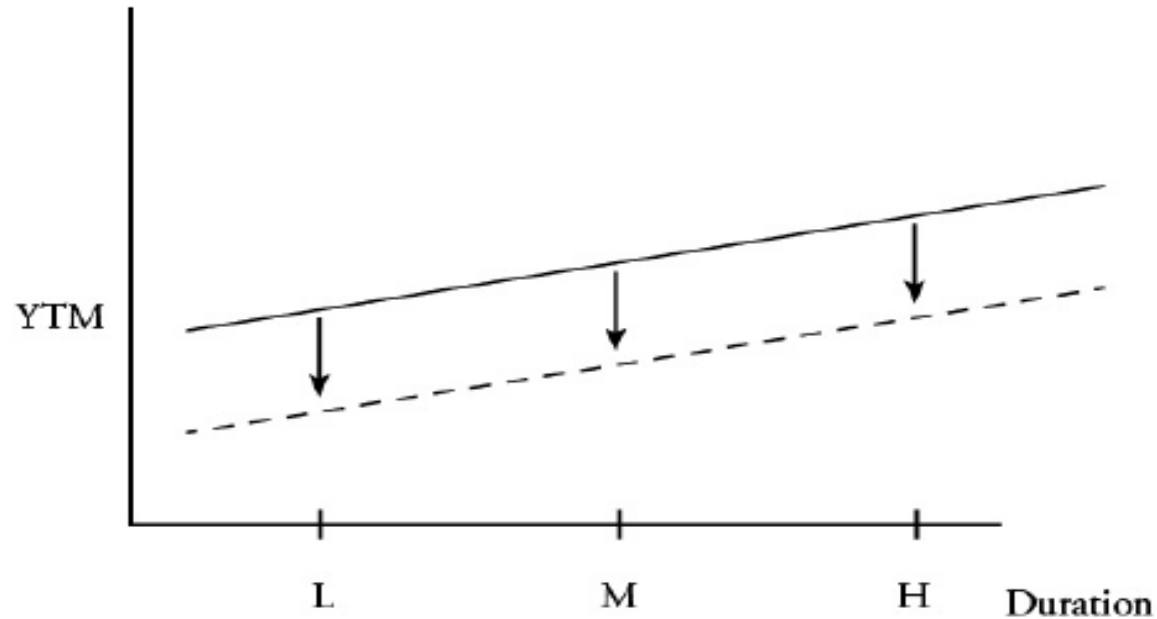


PARALLEL DECREASE IN YIELDS

- For a large parallel decrease in the curve, the immediate increase in barbell value will exceed the increase in the PVL due to the greater convexity effect (WHICH IS ALWAYS POSITIVE IRRESPECTIVE OF THE DIRECTION OF SHIFT).
- With the parallel decrease, the new portfolio IRR will decrease by basically the same amount as the decrease in discount rate for the PVL.

- In other words, the future rate of increase in A and L are still the same, but starting from a new PVA that is relatively higher than the new PVL , the FVA will exceed the FVL .

PARALLEL DECREASE IN YIELDS



EXAMPLE

- Consider the barbell strategy consisting of two bonds S & L with equal money weights (INR 1,044) of 1 year and 9 year ZCBs both with YTM's of 15% p.a. for immunizing a ZCB liability M of INR 4,200 due at $t=5$ years valued at INR 2000 at $t=0$. Assume that there is a instantaneous parallel downward shift of the yield curve by 2%. Evaluate the performance of the strategy.

SOLUTION

- Initial value of liability = 2000
- IRR (YTM) of liability: $\left(\frac{4200}{2000}\right)^{0.2} - 1 = 16\%$
- Initial investment in barbell: $\frac{4200}{1.15^5} = 2088$
- Redemption value of S bond at 15% YTM at one year = $1,044 \times 1.15 = 1200.60$
- Redemption value of L bond at 15% YTM at nine years = $1,044 \times 1.15^9 = 3672.66$

- After decline in interest rates by 2%
- $P_S^1 = \frac{1200.60}{1.13} = 1062.48$
- $P_L^1 = \frac{3672.66}{1.13^9} = 1222.57$
- $P_{BARBELL}^1 = P_S^1 + P_L^1 = 2285.05$
- $P_M^1 = \frac{4200}{1.14^5} = 2181.35$
- Clearly $P_{BARBELL}^1 > P_M^1$

- In this problem, the initial IRR of the barbell is 15% & the liability is 16% and the shift is 2% downwards uniformly. Hence, value of barbell & liability at $t=5$ years:
- Value of liability = $2181.35 \times 1.14^5 = 4200.00$
- Value of barbell = $2285.05 \times 1.13^5 = 4210.06$